**EXAM # 2** (ECE 493/593 Telehealthcare Engineering; Nov 15, 2012)

**Name:**

* Only covers Bayesian Theorem; 50-min test; 10% of final grade.

**Q1 (20%).** Suppose we obtain the following data points from a Gaussian Distribution:

{ 10, 20, 25, 15, 30, 16, 40, 12 }

(1) (10 pts) What is the likelihood?

(2) (10 pts) Deduce the math of using ML (maximum likelihood) to seek the mean µ of the Gaussian Distribution; Then use the above data to find its value.

**Q2 (20%).** Suppose we have the following statistics in the history of a city:

|  |  |  |  |
| --- | --- | --- | --- |
| Seasons  Temperature | Spring | Summer | Fall |
| Cold | 25 times | 1 time | 6 times |
| Warm | 15 times | 3 times | 23 times |
| Hot | 2 times | 29 times | 3 times |

(1) (3 pts) Joint Probability: What is the probability of P { Season = Summer AND Temp = Warm } ?

(2) (3 pts) Marginal probability: What is the probability of P { Temp = Hot (no matter which season) } ?

(3) (3 pts) Conditional probability: What is the probability of P { Temp = Cold | when Season = Fall }?

(4) (3 pts) Use an example from the above table to explain SUM RULE .

(5) (3 pts) Use an example from the above table to explain PRODUCT RULE .

(6) (5 pts) Use an example from the above table to explain **Bayes' Theorem** .

**Q3 (30%).** Polynomial-based Curve Fitting.

(1) (5 pts) Assume we are given N pairs of data, (x1, t1), (x2, t2), ... , (xn, tn). What is the main purpose of using curve fitting?

(2) (10 pts) Use traditional way , i.e., minimizing the sum of the square errors (between true t 's value and fitted curve value), to seek the polynomial coefficients W = [w0, w1, ..., wk ] (order = k). Just list the math formulas step by step.

(3) (15 pts) Now, please use Maximum Likelihood method to seek the value of W. Again, provide detailed math deduction procedure step by step.

**Q4 (30%).** For the above polynomial-based curve fitting case,

(1) (7 pts) For the above sub-question (2), we know that simple *sum of square errors* can cause overfitting issues. Can you **draw a figure** to explain overfitting? What will happen to the coefficients W if overfitting occurs?

(2) (8 pts) Following the above question, how do we improve *sum of square errors* by adding an item (called regularization)? Provide such a formula.

(3) (15 pts) How do we solve the above issue by using MAP (Maximum a Posterior)? Please provide all formulas step by step. Hints: you can assume the prior P(W) fits a Gaussian distribution. That is:

