Hardware Components: Sensors, Actuators, Converters +

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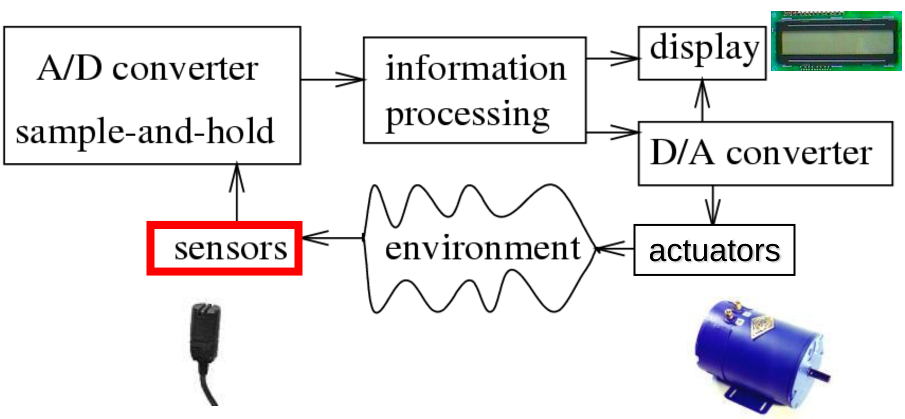
CS/ECE 561 Fall 2012

Simplified Block Diagram



Sensors and Actuators

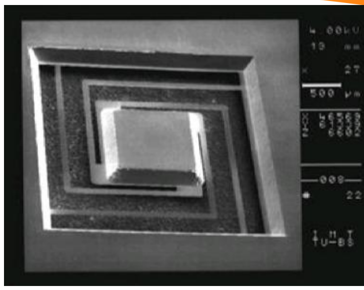
- **Sensors:**
  - Capture physical stimulus (e.g., heat, light, sound, pressure, magnetism, or other mechanical motion)
  - Typically generate a proportional electrical current
  - May require analog interface
- Actuators
  - Convert a command to a physical stimulus (e.g., heat, light, sound, pressure, magnetism, or other mechanical motion)
  - May require analog interface

Sensors

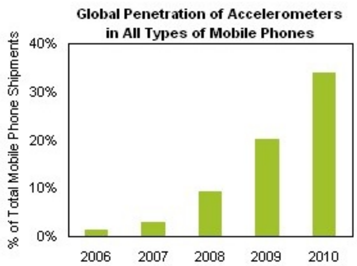
- Processing of physical data starts with capturing data from sensors
- Sensors can be designed for virtually every physical stimulus
  - heat, light, sound, weight, velocity, acceleration, electrical current, voltage, pressure, ...
- Many physical effects used for constructing sensors.
  - law of induction
    - generation of voltages in an electric field
  - light-electric effects; magnetic effects; …

Example: Acceleration Sensor

- **MEMS device**
  Microelectromechanical systems (~ 1 to 100 µm)
- **Small mass in center**
- **When accelerated:**
  - Mass displaced from center
  - Resistance of wires connected to mass change
  - Detect change in resistance and model acceleration

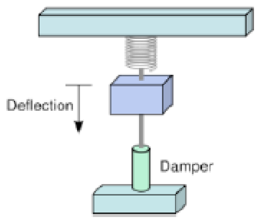


iPhones have MEMS accelerometers

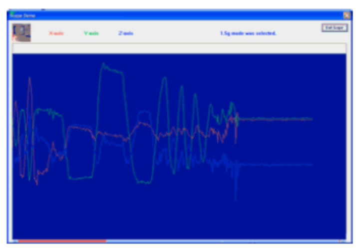


Example: Acceleration Sensor

- **Alternative implementation:**
  - accelerometer measures distance between a plate fixed to the platform and one attached by a spring and damper
  - measurement is typically done by measuring capacitance

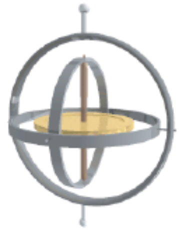


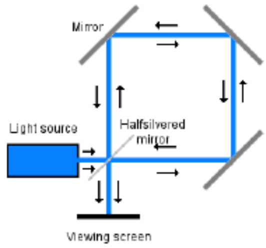
- **Many uses!**
  - Navigation, orientation, drop detection, image stabilization, airbag systems
- **Challenges**
  - Vibration
  - Nonlinearities in the spring or damper
  - Separating tilt from acceleration



Freescale ZStar is a development kit with a 3-axis accelerometer, wireless link, USB dongle, and demo software.

Measuring Changes in Orientation: Gyroscopes

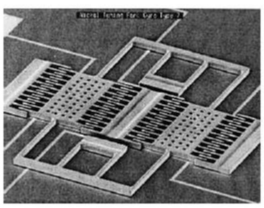




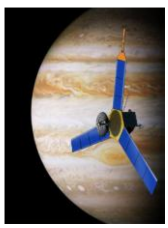
Optical gyros: Leverage the Sagnac effect, where a laser light is sent around a loop in opposite directions and the interference is measured. When the loop is rotating, the distance the light travels in one direction is smaller than the distance in the other. This shows up as a change in the interference.

Example: MEMS Gyroscope

MEMS gyroscope - orientation



Vibrating-Wheel Gyroscope



MEMS gyroscope for satellite positioning

Digital cameras using MEMS gyroscopes for image stabalization

Wii's controller uses MEMS sensors, MEMS accelerometers and MEMS gyroscope (MotionPlus)

	2002 ($ 000,000)	2007 ($ 000,000)
Microfluidics	1,404	2,241
Optical MEMS	702	1,826
RF MEMS	39	249
Other actuators	117	415
Inertial sensors	819	1,826
Pressure sensors	546	913
Other sensors	273	830
Total	3,900	8,300

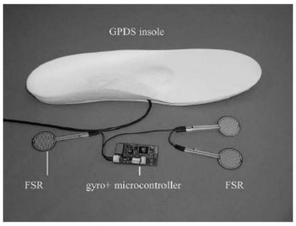
The forecasted compounded annual growth rate (CAGR) between 2002 and 2007 is 16%.
(*Source:* [6]).

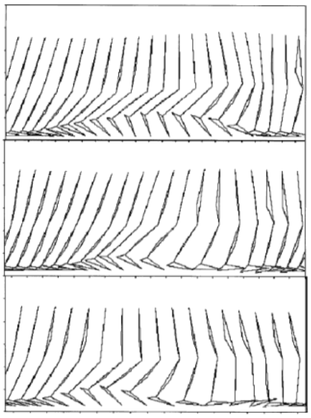
Inertial Navigation Systems

- Combinations of:
  - GPS (for initialization and periodic correction).
  - Three axis gyroscope measures orientation.
  - Three axis accelerometer, double integrated for position after correction for orientation.
- Typical drift for systems used in aircraft have to be:
  - 0.6 nautical miles per hour
  - tenths of a degree per hour
- Good enough? It depends on the application!

Example: Gait-Phase Detection sensor Embedded in a Shoe Insole

- **Measures the angular velocity of the foot**
- **Used to activate a functional electrical stimulator (FES) attached to the foot to activate nerves**
- **Over 96% accuracy**

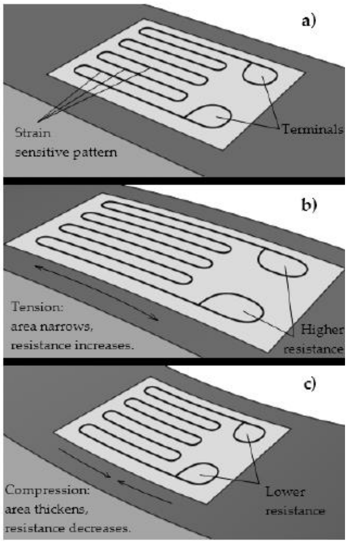




Example: Strain Gauges

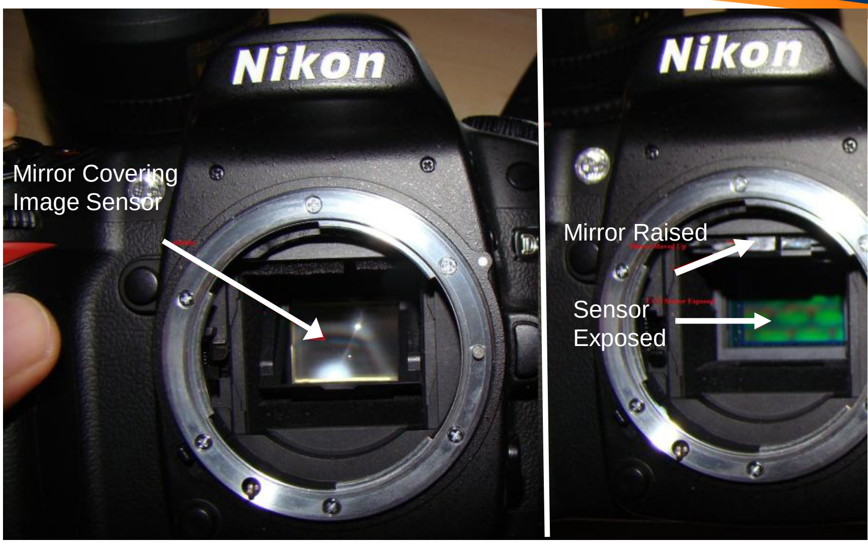
Strain Gauges

Mechanical strain gauge used to measure the growth of a crack in a masonry foundation. This one is installed on the Hudson-Athens Lighthouse. Photo by Roy Smith, used with permission.



11

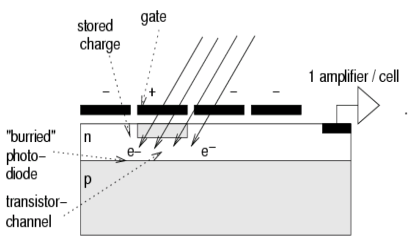
Image Sensors

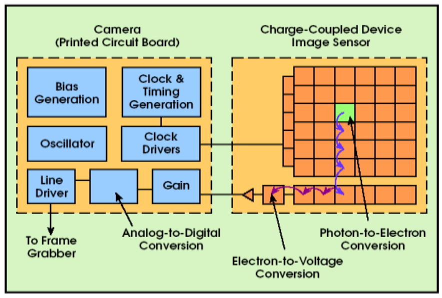


12

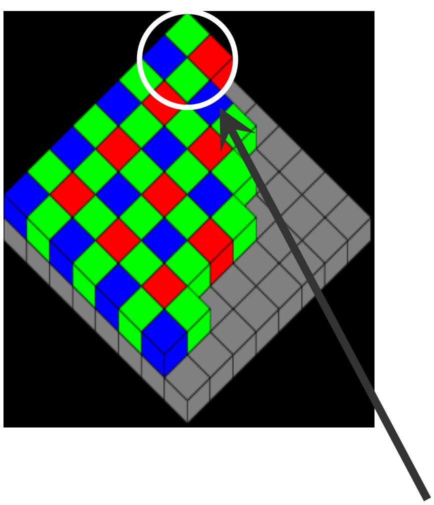
Example: Charge-coupled devices (CCD)

- Image is projected by lens on a capacitor array (photoactive)
- Each capacitor accumulates an electric charge
  - proportional to the light intensity at that location
- Next, a control circuit causes each capacitor to 'shift' its contents to its neighbor
- The last capacitor in the array dumps its charge into a charge amplifier, which converts the charge into a voltage





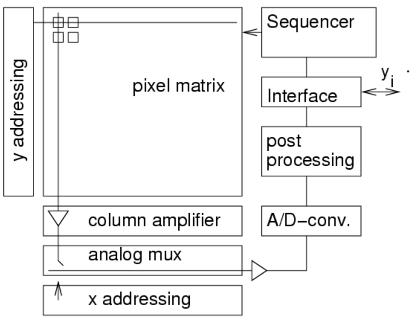
How CCD Sensors Record Color

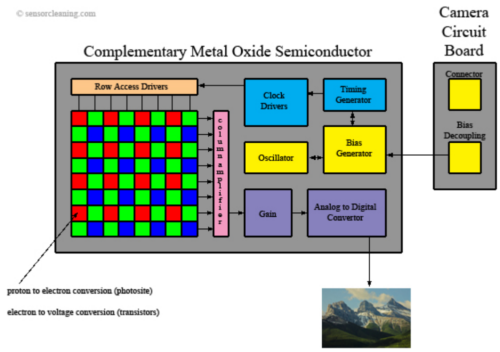


- Each CCD cell in the CCD array produces a single value independent of color.
- To make color images, CCD cells are organized in groups of four cells (making one pixel) and a Bayer Filter is placed on top of the group to allow only red light to hit one of the four cells, blue light to hit another and green light to hit the remaining two.
- The reasoning behind the two green cells is because the human eye is more sensitive to green light and it is more convenient to use a 4 pixel filter than a 3 pixel filter (harder to implement) and can be compensated after a image capture with something called white balance.
- Ex. A Bayer filter applied to the underlying CCD pixel

Example: CMOS Image Sensors

- Based on standard production process for CMOS chips, allows integration with other components
- Sensor array made of photo detector and amplifier transistors at each pixel
- Many of the photons hitting the chip hit the transistors instead of the photodiode – lower light sensitivity



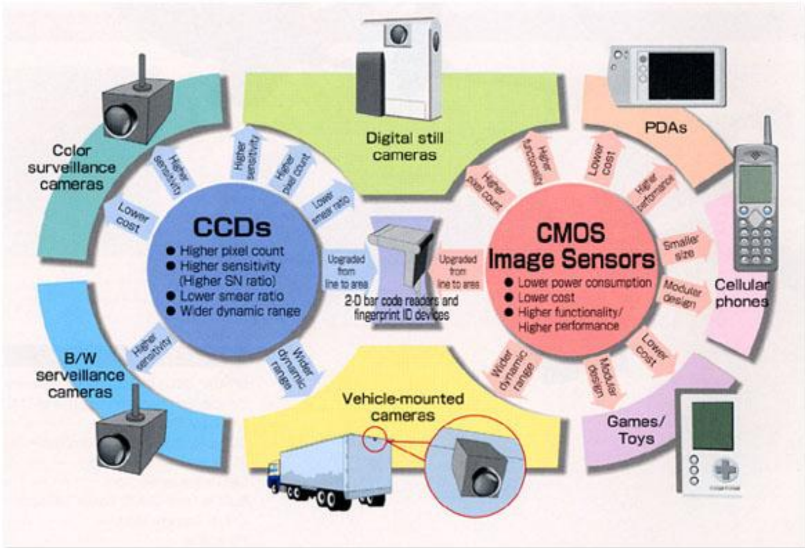


Comparison CCD/CMOS sensors

Property	CCD	CMOS
Signal/noise ratio (SNR)	Excellent	Medium
Dark current	Very low	Medium
Technology optimized for	Optics	VLSI technology
Technology	Special	Standard
Smart sensors?	No, no logic or A/D converters on chip	Logic elements on chip
Access	Serial	Random
Interface	Complex	Simple, single VDD

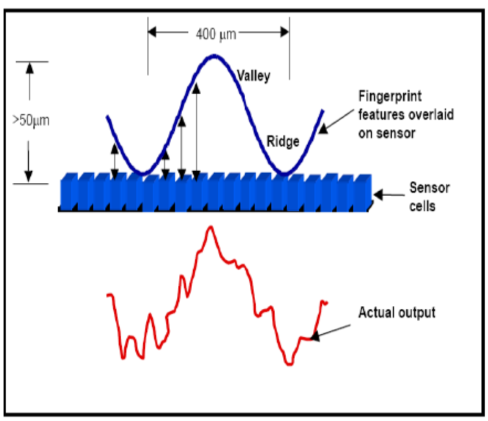
Source: B. Dierickx: CMOS image sensor concepts. Photonics West 2000 Short course (Web)

Comparison CCD/CMOS sensors



Example: Biometric Sensors

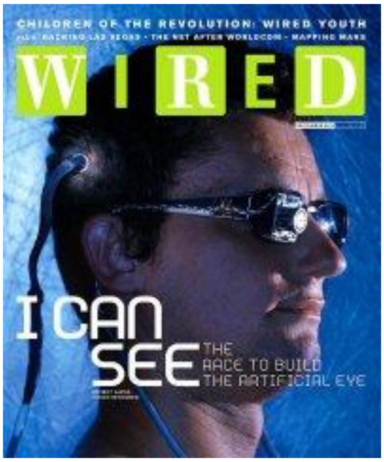
Example: Fingerprint sensor (© Siemens, VDE):

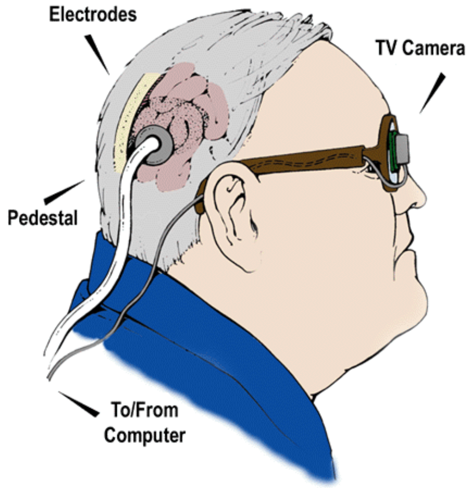


Matrix of 256 x 256 elem.
Voltage ~ distance.
Resistance also computed.

No fooling by photos and wax copies!

Example: Artificial eyes





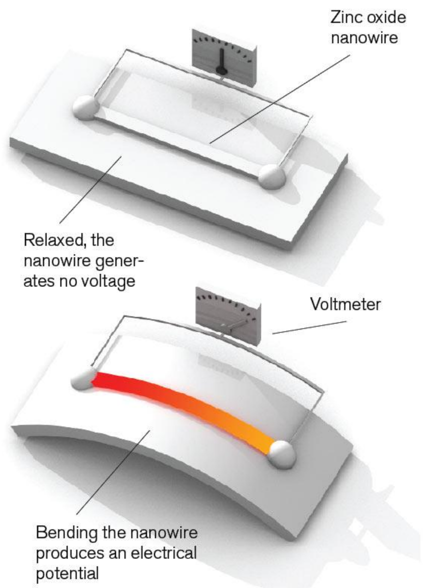
© Dobelle Institute
(www.dobelle.com)

Example: Artificial eyes (2)

- **Retinal implants (BRI Project)**
  - array of electrodes to stimulate light sensing cells that do not work
  - camera on pair of eyeglasses captures image
  - image processed by a microcontroller to produce a simplified picture
  - picture wirelessly beamed to the implant, which activates 15 electrodes inside the eye (to create a15-pixel image)
    - **Goal: 1000+ pixel images**
  - implant also receives power wirelessly from the microcontroller
  - electronics housed within a waterproof titanium case similar to those used for heart pacemakers

© Boston Retinal Implant Project
(http://www.bostonretinalimplant.org)

Example: Self Powering Sensors



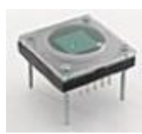
- **Nanoscale sensors**
  - piezoelectric
  - zinc oxide nanowires
  - subtle movements can bend these wires
    - sound waves, wind, even turbulence of blood flow over an implanted device
  - Uses:
    - Hearing aids, bone density loss monitors, “electric current from clothes”

21

Other examples of sensors

- Heart monitoring sensors
  - “Managing Care Through the Air”
    - » IEEE Spectrum Dec 2004
- Rain sensors for wiper control
  - High-end autos
- Pressure sensors
  - Touch pads/screens
- Proximity sensors
  - Collision avoidance
- Vibration sensors
- Smoke sensors
  - Based on the diffraction of light waves in the presence of smoke
- Thermal sensors (thermistors)
  - SARS detection (“high fever”)





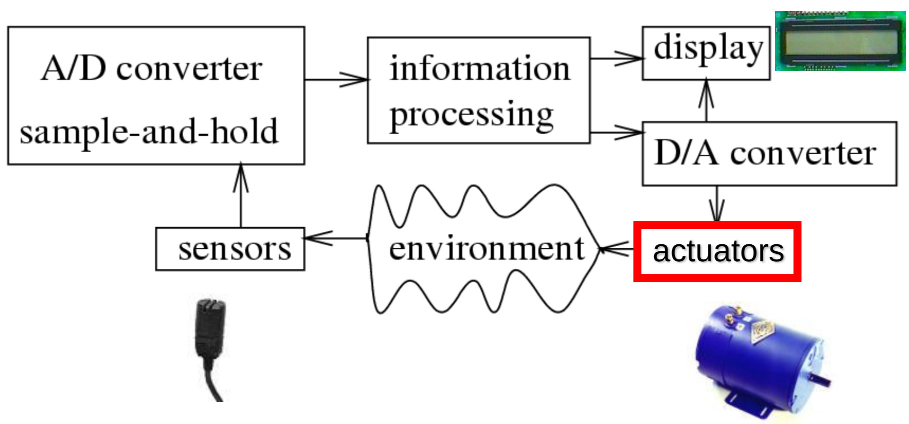


22

Design Issues with Sensors

- Calibration
  - Relating measurements to the physical phenomenon
  - Can dramatically increase manufacturing costs
- Nonlinearity
  - Measurements may not be proportional to physical phenomenon
  - Correction may be required
  - Feedback can be used to keep operating point in the linear region
- Sampling
  - Aliasing
  - Missed events
- Noise
  - Analog signal conditioning
  - Digital filtering
  - Introduces latency

Simplified Block Diagram



Sensors and Actuators

- **Sensors:**
  - Capture physical stimulus (e.g., heat, light, sound, pressure, magnetism, or other mechanical motion)
  - Typical generate a proportional electrical current
  - May require analog interface
- **Actuators**
  - Convert a command to a physical stimulus (e.g., heat, light, sound, pressure, magnetism, or other mechanical motion)
  - May require analog interface

solenoid

speaker

laser diode/transistor

dc motor

LED display

25

Actuators

- **Output physical stimulus varies in range and modality**
  - Large (industrial) control actuators
    - Pneumatic systems: physical motion
  - Optical output
    - IR, LEDs, displays, etc.
  - Motor controllers
    - DC, stepper, servo, …
  - Sound
    - Loudspeakers, etc.
  - List goes on…..

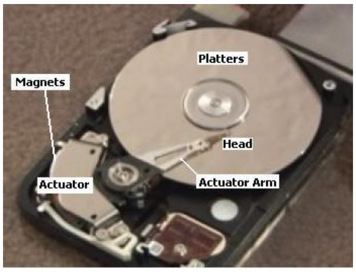
26

Stepper Motor Controller

- **Stepper motor: rotates fixed number of degrees when given a "step" signal**
  - In contrast, DC motor simply rotates when power applied, and coasts to stop
- **Rotation achieved by applying specific voltage sequence to coils**
  - Controller greatly simplifies this
- **Stepper motors used commonly in hard drives (traditionally)**
  - Head motor (stepper), spindle motor (DC)

Stepper Motor Controller

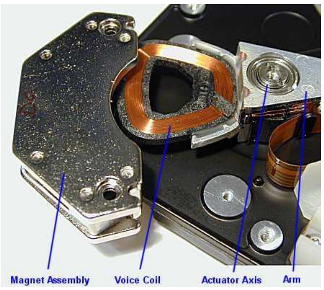
Sequence	A	B	A'	B'
1	+	+	-	-
2	-	+	+	-
3	-	-	+	+
4	+	-	-	+
5	+	+	-	-




```
sbit SM_A, SM_B, SM_AP, SM_BP; // ports
int curr_pos; // tells us the current step position
void reset() {  // must be called to synchronize
  curr_pos = 0;
  for(int i=0; i<4; i++) {
    move_one_step(0);
  }
}
void move_one_step(int dir/*0=CW,1=CCW*/) {
  const int SM_TBL[4][4] = {
  1, 1, 0, 0,    0, 1, 1, 0,    0, 0, 1, 1,    1, 0, 0, 1 };
  curr_pos = (curr_pos + (dir == 0 ? +1 : +3)) % 4;
  SM_A = SM_TBL[curr_pos][0];
 SM_B = SM_TBL[curr_pos][1];
 SM_AP = SM_TBL[curr_pos][2];
 SM_BP = SM_TBL[curr_pos][3];
 ms_delay(50);
}
```

Servo Motor Controller

- **Actuator in a modern hard disk uses**
  - a device called a *voice coil* to move the head arms in and out over the surface of the platters
  - a closed-loop feedback system called a *servo system* to dynamically position the heads directly over the data tracks
    - works using electromagnetic attraction and repulsion
- **How it works:**
  - Coil is wrapped around a metal protrusion on the end of the set of head arms
  - This is mounted within an assembly containing a strong permanent magnet
  - When current is fed to the coil, an electromagnetic field is generated that causes the heads to move
  - By controlling the current, the heads can be told to move in or out much more precisely than using a stepper motor

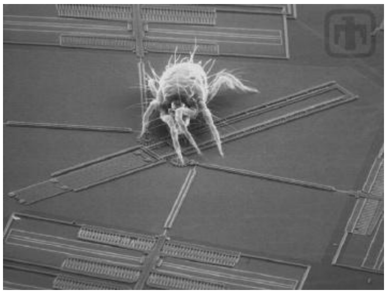


29

MEMS Actuators

Microelectromechanical systems (~ 1 to 100 µm)
Huge variety of actuators and output devices.
Very small devices with low power consumption
E.g., applications: space telescopes and DLP projectors (moving micromirrors), miniaturized drug delivery inside human body (bio-MEMS), accelerometers, gyroscopes

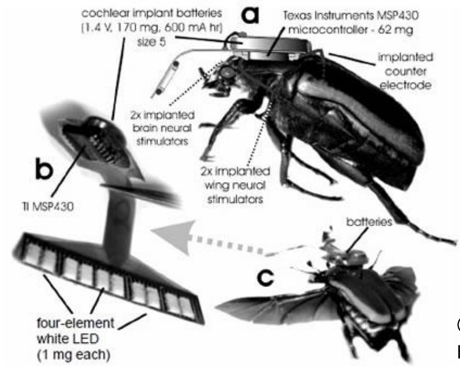
Silicon Gear and Chain – Chain Links 50 µm Apart



Spider Mite Crawling on Micro-Mirror Device

30

Actuators: The Future?



- **Cyborg Beetle**
  - giant flower beetle with implanted processor , microbattery, electrodes
  - electrodes deliver electrical jolts to its brain and wing muscles
  - flight can be wirelessly controlled
    - e.g. take off, turn, stop midflight.

© Berkeley Labs

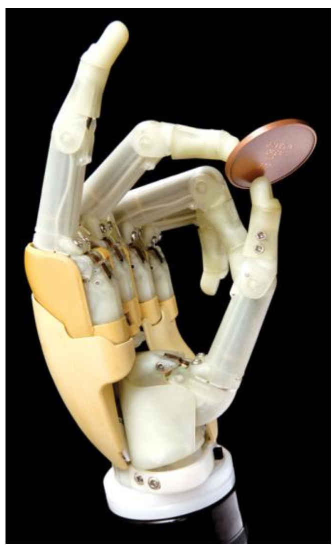
http://www.technologyreview.com/computing/22039/?a=f

31

Touch Bionics iLIMB Actuator

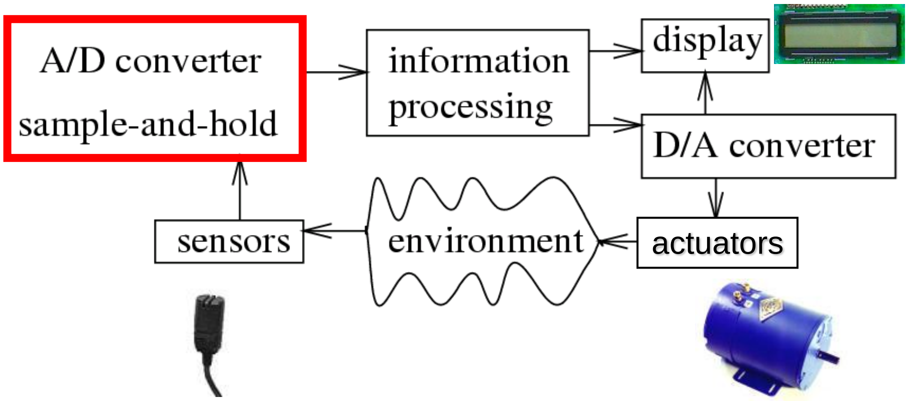
It's got an embedded computer, a rechargeable battery, and five small dc motors. It costs US $18 500. And it can do things most other prosthetic hands just can't, like grabbing a paper cup without crushing it, turning a key in a lock, and pressing buttons on a cellphone.

The fingers of Touch Bionics' iLIMB Hand are controlled by the nerve impulses of the user's arm, and they operate independently, adapting to the shape of whatever they're grasping. The hand can also do superhuman tricks, like holding a very hot plate or gripping an object tirelessly for days. A skin-tone covering gives the bionic hand a lifelike look, but some customers refer semitransparent models, to proudly flaunt their robotic hands. "They like the Terminator look," says Touch Bionics CEO Stuart Mead. IEEE Spectrum, Oct. 2007.



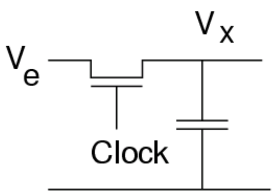
32

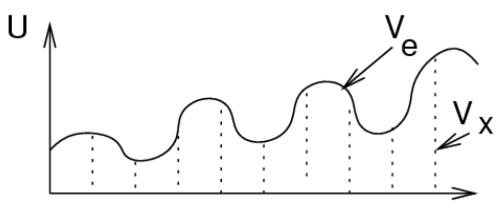
Simplified Block Diagram



33

Discretization of Time





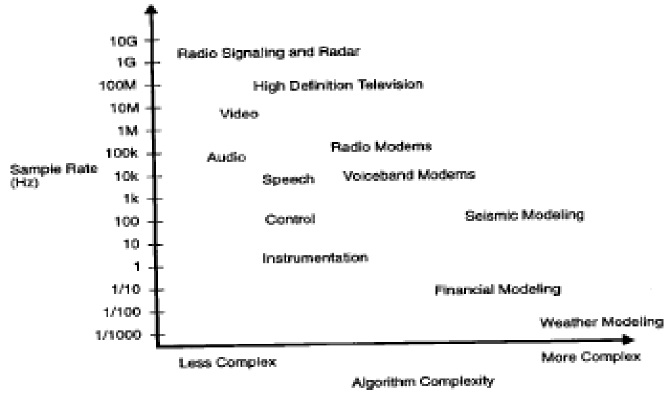
Sample and hold circuit

Sampling: how often the signal is converted?
Quantization: how many bits used for samples?

34

Sampling

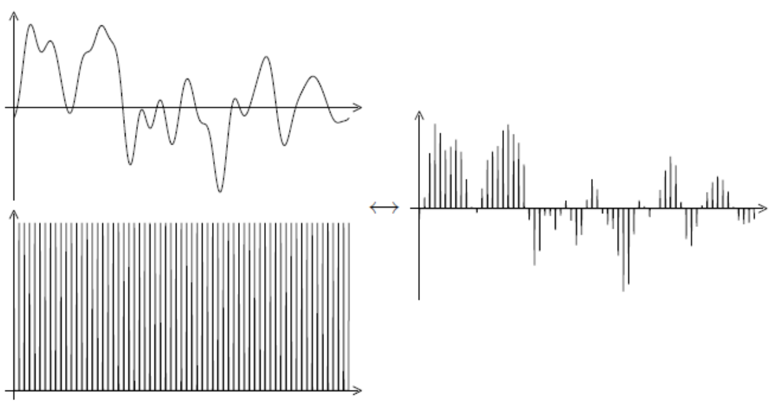
- **Sampling: how often is the signal converted (sampled) to represent original signal?**
  - Twice as high as the highest frequency signal present in the input
    - **Nyquist sampling theorem**
  - As much as 10 to 20 times for even better results
- **Typical Sampling Rates**



35

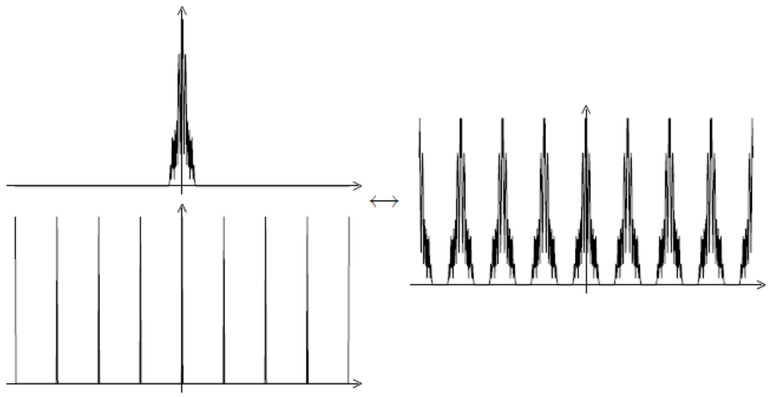
Sampling Process – Time Domain

- **Sampling = multiplying with comb function**



36

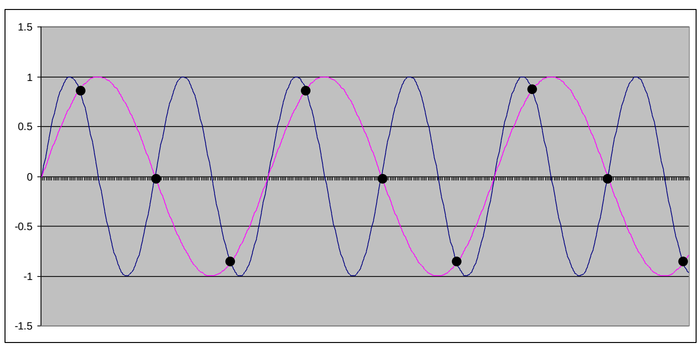
Sampling Process – Freq Domain



Aliasing

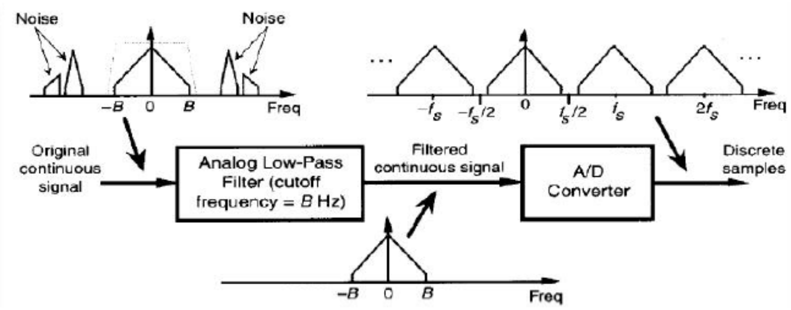
- **Aliasing: erroneous signals, not present in analog domain, but present in digital domain**

Signal freq: 5.6 Hz
Sampling freq: 9 Hz



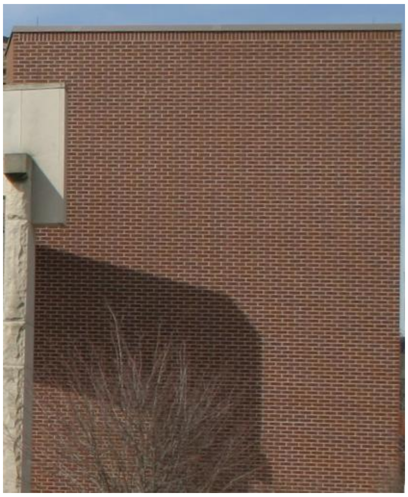
Preventing Aliasing

- Sample at higher than necessary rate
- Use anti-aliasing filters
  - Helps eliminate high frequency noise that can cause aliasing of lower frequency input signal



Examples of Aliasing in computer graphics

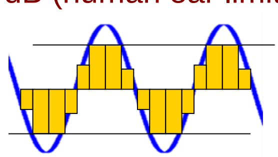
Original



Sub-sampled, no filtering

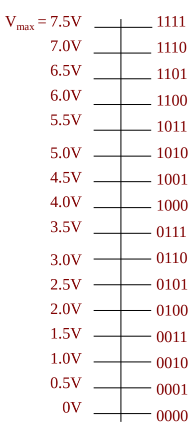
Quantization

- Sample precision - the resolution of a sample value
- Quantization depends on the number of bits used to measure the height of the waveform.
- Audio formats are described by sample rate and quantization.
  - Voice quality - 8 bit quantization, 8000 Hz mono(8 Kbytes/sec)
  - 22kHz 8-bit mono (22kBytes/s) and stereo (44Kbytes/sec)
  - CD quality - 16 bit quantization, 44100 Hz linear stereo (196 Kbytes/s)
- Quantization: how many bits used to represent a sample?
  - Sufficient to provide required dynamic range
    - 16-bit A/D ➔ $20 \times \log_{10}(2^{16}) = 96$ dB (human ear limit)
  - Clipping: input signal beyond the dynamic range

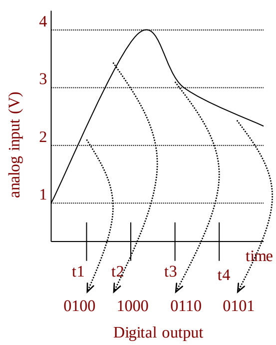


41

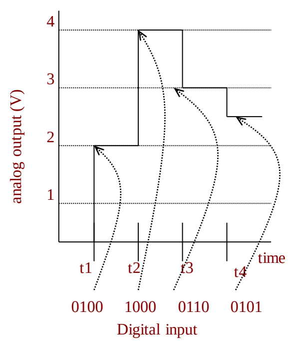
Analog-to-Digital Converter



proportionality



analog to digital

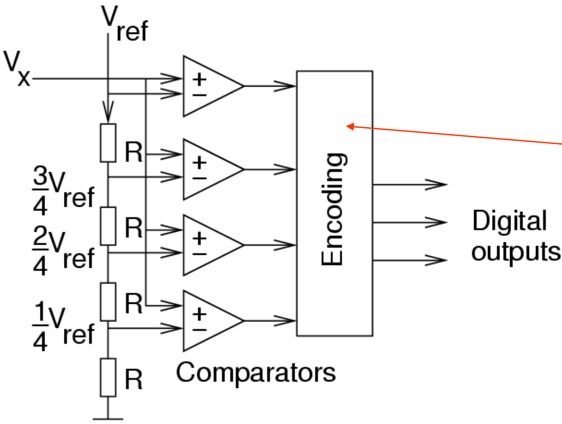


digital to analog

42

A/D Converters: Flash A/D Converter

Digital computers require digital form of physical values
A/D-conversion; many methods with different speeds.

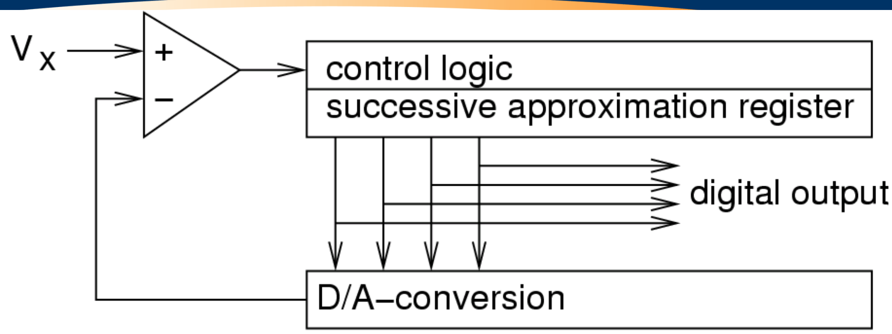


- **Parallel comparison with reference voltage**
- **Speed: *O(1)***
- **HW complexity: *O(n)***
  - *n*= # of distinguished voltage levels

Encodes input number of most significant ‘1’ as an unsigned number, e.g.
“1111” -> “100”,
“0111” -> “011”,
“0011” -> “010”,
“0001” -> “001”,
“0000” -> “000”
(Priority encoder).

43

A/D Converters: Successive Approximation

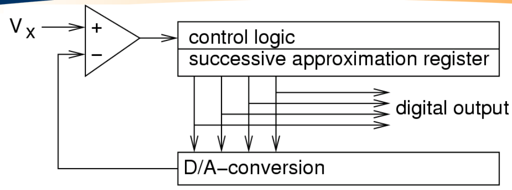


Key idea: binary search:
Set MSB='1'
if too large: reset MSB
Set MSB-1='1'
if too large: reset MSB-1
…..

Speed: *O(log(n))*
Hardware complexity: *O(1)*
with *n*= # of distinguished voltage levels;
slow, but high precision possible.

44

Successive Approximation

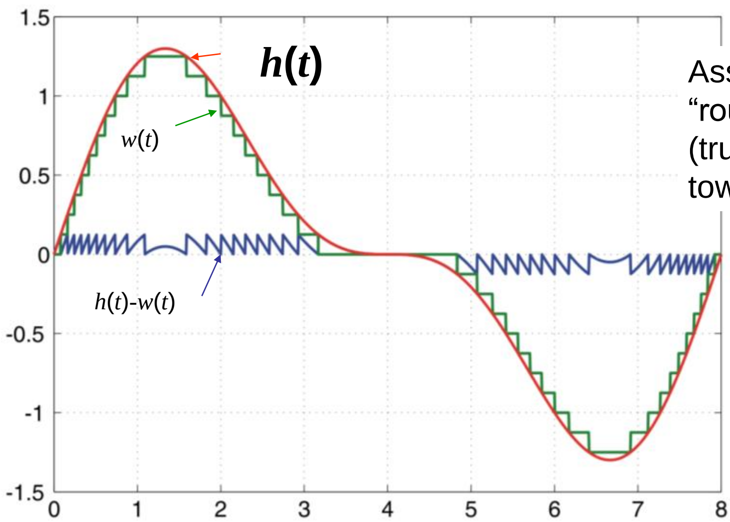


Given an analog input signal whose voltage should range from 0 to 15 volts, and an 8-bit digital encoding, calculate the correct encoding for 5 volts.

½(V_{max} + V_{min}) = 7.5 volts V_{max} = 7.5 volts.	0	0	0	0	0	0	0	0
½(7.5 + 0) = 3.75 volts V_{min} = 3.75 volts.	0	1	0	0	0	0	0	0
½(7.5 + 3.75) = 5.63 volts V_{max} = 5.63 volts	0	1	0	0	0	0	0	0
½(5.63 + 3.75) = 4.69 volts V_{min} = 4.69 volts.	0	1	0	1	0	0	0	0
½(5.63 + 4.69) = 5.16 volts V_{max} = 5.16 volts.	0	1	0	1	0	0	0	0
½(5.16 + 4.69) = 4.93 volts V_{min} = 4.93 volts.	0	1	0	1	0	1	0	0
½(5.16 + 4.93) = 5.05 volts V_{max} = 5.05 volts.	0	1	0	1	0	1	0	0
½(5.05 + 4.93) = 4.99 volts	0	1	0	1	0	1	0	1

45

Quantization Noise



Assuming “rounding“ (truncating) towards 0

47

How Fast should ADC be?

Applications	Appro. No of conversions per second	required conversion time
monitor and control	1 - 1000	1s - 1 ms
telephone voice	8,000	125 μs
CD-quality audio	85, 42.5, 21.3 K	50 - 12 μs
Video	$1-10 \times 10^6$ $3 \times 1-10 \times 10^6$	100 ns - 1 μs
radar	$100-1000 \times 10^6$	1 - 10 ns

Signal Processing

- **Any interesting embedded system has to process some input signals and generate some output signals**
  - We use the term *signal* in a general way
- **Digital devices process signals in digital form**
  - A uniformly sampled stream of data spread in time (e.g., audio) or space (e.g., image)

Signals and Systems

- Signal: set of information and data
  - E.g., audio, video, radio,…
  - Continuous vs. discrete-time
  - Analog and digital
- System: entity that *transforms* input signals to output signals
  - Linear and non-linear
  - Constant parameter and time-varying parameter
  - Causal and non-causal
- Digital Signal Processing (DSP)
  - Digitize signals and process them in the digital domain
    - In software (e.g., DSPs, ASIPs, GPPs) or hardware (ASIC, FPGA)

50

Analog vs. Digital

Analog

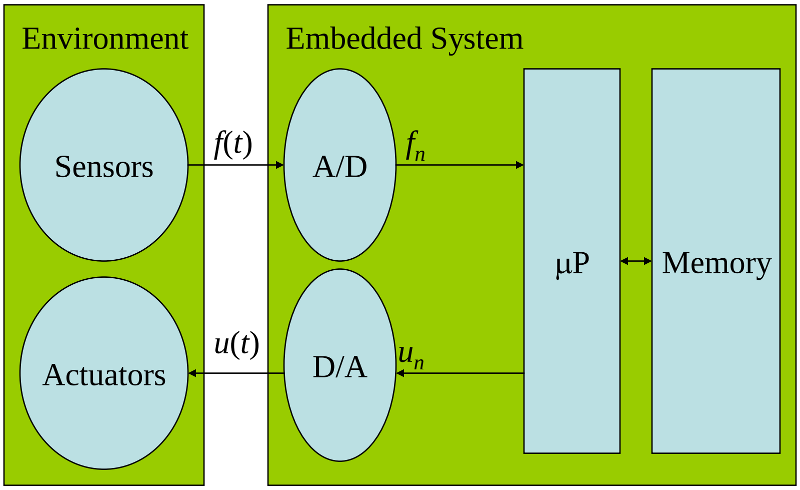
- Good
  - High bandwidth
  - High resolution
  - Specific control functions are available as off-the-shelf ICs
  - Analysis and design methods are well-known
- Bad
  - Temperature drift
  - Component aging
  - Sensitive to noise
  - Hardware design
  - Can implement simple designs only

Digital

- Good
  - Programmable solution
  - Less sensitive to environment
  - Can implement advanced control algorithms
  - Capable of self-tuning, adaptive control, and nonlinear control functions
- Bad
  - Data converter is required.
  - Analysis and design methods are more complex
  - Sampling & quantization error
  - Computation delay limits the system bandwidth

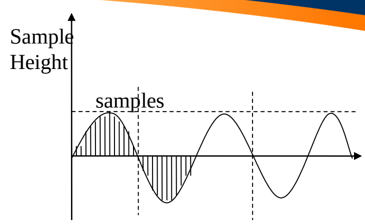
51

General DSP Architecture



52

Signal Processing

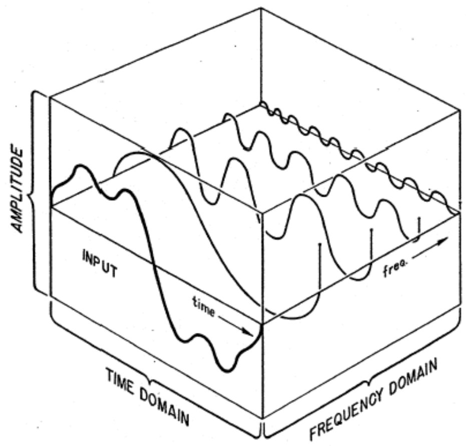


- **Digital signal $S_0, S_1, S_2 \ldots S_{n\text{-}1}$**
- **What can we do with it?**
  - Transpose: e.g., $Z_i = S_i + K$
  - Amplify: e.g., $Z_i = S_i \times \alpha$
  - Compose: e.g., $Z_i = (S^1_i \times \alpha^1 + K^1) + (S^2_i \times \alpha^2 + K^2)$
  - Filter: e.g, $Z_i = (S_i + S_{i+1}) / 2$
  - Compress: e.g., using Huffman codes
  - Archive, match against database, etc.
- **Or, process after converting to frequency domain**
  - Spectral analysis

53

Time Domain

- **Any continuous *periodic* time varying signal can be represented as the sum of cosine functions of different amplitude and frequency**
  - E.g., input signal captured as the sum of 4 cosine functions
  - Captured as a Trigonometric Fourier Series (continuous)

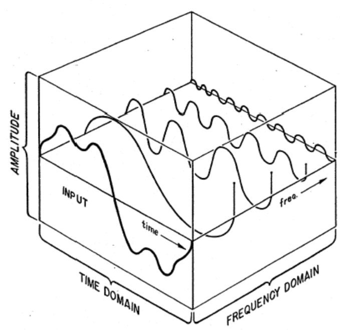


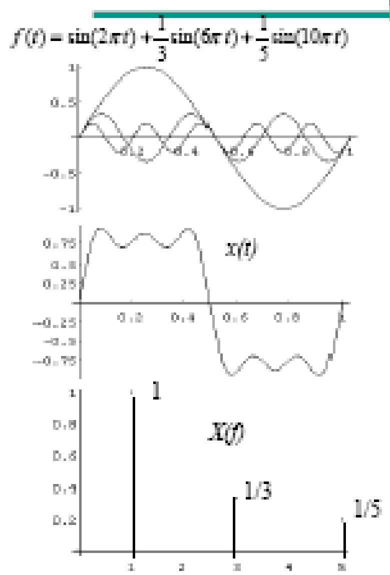
$$f(t) = a_0 + \sum_{k=1}^{\infty} a_k \cos(2\pi f_0 kt) + \sum_{k=1}^{\infty} b_k \sin(2\pi f_0 kt)$$

$$= C_0 + \sum_{k=1}^{\infty} C_k \cos(2\pi f_0 kt + \theta_k)$$

54

Time and Frequency Domain





- **Signal can be represented in frequency domain**
  - Discrete values, representing amplitude at each frequency for the periodic signal
- **How about *aperiodic* signals?**
  - Linear function of infinite number of sinusoidal functions…

55

(Continuous) Fourier Transform (FT)

- Represent periodic functions
- Input: $h(t)$ *continuous time domain* signal
- Output: $H(f)$ *continuous frequency domain* signal
- $H(f)$ is represented by complex number $a+b\times j$
- Amplitude: $\sqrt{(a^2+b^2)}$
- Phase-shift: $\arctan(b/a)$

$$H(f) = \int_{-\infty}^{\infty} h(t) e^{-j2\pi ft} dt$$

$$h(t) = \int_{-\infty}^{\infty} H(f) e^{j2\pi ft} df$$

$$e^{j\omega} = \cos(\omega) + j\sin(\omega)$$

56

Discrete Fourier Transform (DFT)

- Input: h_t discrete time domain signal
- Output: H_f discrete frequency domain signal
- H_f is represented by complex number $a+b\times j$
- Amplitude: $\sqrt{(a^2+b^2)}$
- Phase-shift: $\arctan(b/a)$
- DFT is computationally expensive
  - There is a fast software implementation (FFT)

$$H_f = \frac{1}{N} \sum_{t=0}^{N-1} h_t e^{-j\frac{2\pi ft}{N}} \quad \text{DFT}$$

$$h_t = \sum_{f=0}^{N-1} H_f e^{j\frac{2\pi tf}{N}} \quad \text{IDFT}$$

Euler's Eq.

$$e^{j\omega} = \cos(\omega) + j\sin(\omega)$$

57

Discrete Cosine Transform

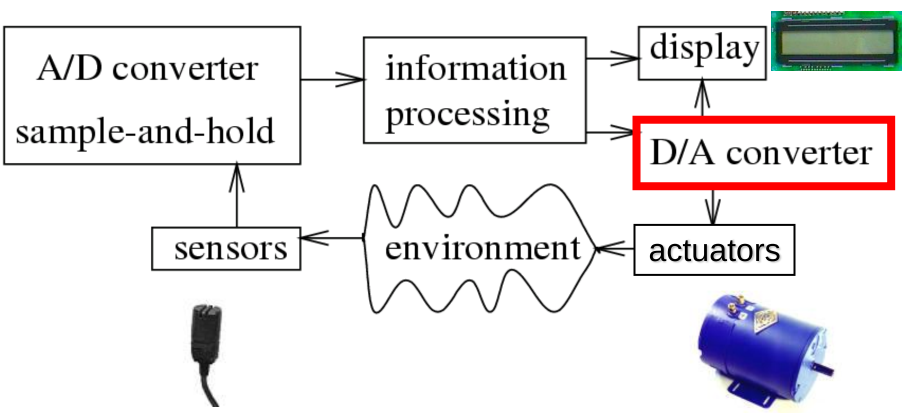
- **A special version of the DFT**
  - Used in JPEG and MPEG compression
- **Why DCT and not DFT?**

$$\sum_{n=0}^{N-1} f(n) \cdot \cos\left[\frac{\pi k}{N} \cdot \left(n + \frac{1}{2}\right)\right] \quad \text{versus} \quad \sum_{n=0}^{N-1} f(n) \cdot e^{-\frac{2\pi i n k}{N}}$$

- Transforms images efficiently into a compressible form.
- Uses only cos, DFT uses sin and cos (hidden within $e^{-i\varphi}$).
- DCT coefficients are real, DFT coefficients are complex.

58

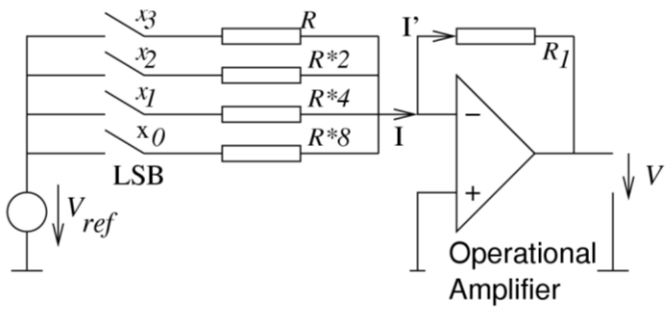
Simplified Block Diagram



59

Digital-to-Analog (D/A) Converters

Various types, can be quite simple, e.g.:

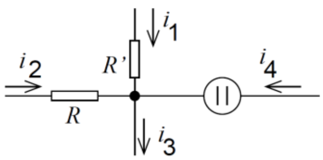


Kirchhoff's junction rule

Kirchhoff's Current Law, Kirchhoff's first rule

- **Kirchhoff's Current Law: At any point in an electrical circuit, the sum of currents flowing towards that point is equal to the sum of currents flowing away from that point.**
  - (Principle of conservation of electric charge)

Example:



$i_1 + i_2 + i_4 = i_3$

$i_1+i_2-i_3+i_4=0$

Formally, for any node in a circuit:

$$\sum_k i_k = 0$$

Count current flowing away from node as negative.

[Jewett and Serway, 2007].

Kirchhoff's loop rule

Kirchhoff's Voltage Law, Kirchhoff's second rule

The principle of conservation of energy implies that:

- **The sum of the potential differences (voltages) across all elements around any closed circuit must be zero**

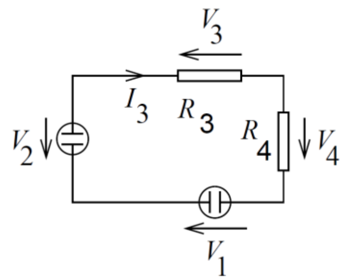
[Jewett and Serway, 2007].

Formally, for any loop in a circuit:

$$\sum_k V_k = 0$$

Count voltages traversed against arrow direction as negative

Example:



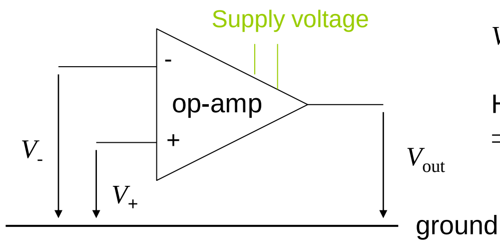
$V_1 - V_2 - V_3 + V_4 = 0$

$V_3 = R_3 \times I_3$ if current counted in the same direction as V_3

$V_3 = -R_3 \times I_3$ if current counted in the opposite direction as V_3

Operational Amplifiers (Op-Amps)

- **Operational amplifiers (op-amps) are devices amplifying the voltage difference between two input terminals by a large gain factor g**



$V_{out} = (V_+ - V_-) \cdot g$

High impedance input terminals
$\Rightarrow$ Currents into inputs ≈ 0

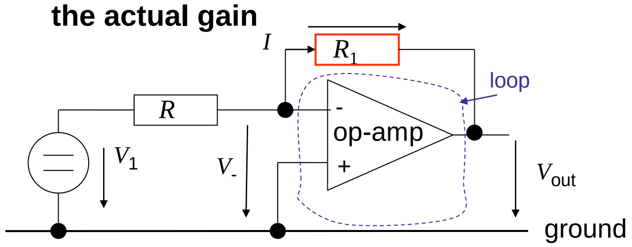
For an **ideal** op-amp: $g \to \infty$

(In practice: g may be around $10^4..10^6$)

Op-amp in a separate package (TO-5) [wikipedia]

Op-Amps with feedback

- **In circuits, negative feedback is used to define the actual gain**



$V_{out} = -g \cdot V_-$ (op-amp feature)

$I \cdot R_1 + V_{out} - V_- = 0$ (loop rule)

$\Rightarrow I \cdot R_1 + -g \cdot V_- - V_- = 0$

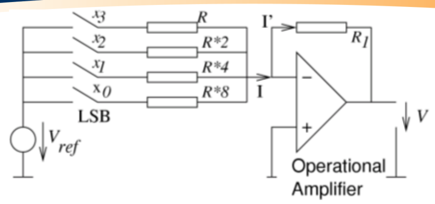
$\Rightarrow (1+g) \cdot V_- = I \cdot R_1$

$$\Rightarrow V_- = \frac{I \cdot R_1}{1+g}$$

$$V_{-,ideal} = \lim_{g \to \infty} \frac{I \cdot R_1}{1+g} = 0$$

V_- is called **virtual ground**: the voltage is 0, but the terminal may not be connected to ground

Output voltage ≈ no. represented by x



Due to Kirchhoff's laws: $I = x_3 \times \frac{V_{ref}}{R} + x_2 \times \frac{V_{ref}}{2 \times R} + x_1 \times \frac{V_{ref}}{4 \times R} + x_0 \times \frac{V_{ref}}{8 \times R}$

$$= \frac{V_{ref}}{R} \times \sum_{i=0}^{3} x_i \times 2^{i-3}$$

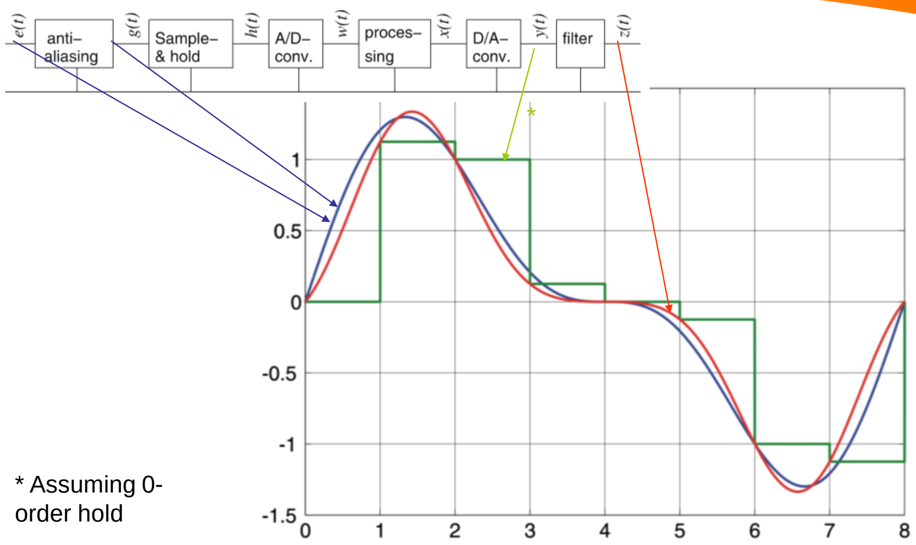
Due to Kirchhoff's laws: $V + R_1 \times I' = 0$

Current into Op-Amp=0: $I = I'$

Hence: $V + R_1 \times I = 0$

Finally: $-V = V_{ref} \times \frac{R_1}{R} \sum_{i=0}^{3} x_i \times 2^{i-3} = V_{ref} \times \frac{R_1}{8 \times R} \times nat(x)$

(Attempted) reconstruction of input signal



* Assuming 0-order hold

Limitations

$$z(t) = \sum_{s=-\infty}^{\infty} \frac{y(t_s) sin \frac{\pi}{T_s}(t - t_s)}{\frac{\pi}{T_s}(t - t_s)}$$

- Actual filters do not compute *sinc*( )
  In practice, filters are used as an approximation.
  Computing good filters is an art itself!
- All samples must be known to reconstruct $e(t)$ or $g(t)$.
  ☞ Waiting indefinitely before we can generate output!
  In practice, only a finite set of samples is available.
- Quantization noise cannot be removed.